

$$45 \lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}}$$

Jan 22-9:24 AM

8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty \quad f \text{ is faster than } g$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0 \quad g \text{ is faster than } f$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = L \quad f, g \text{ same}$$

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$y = x, y = 2x \quad \lim_{x \rightarrow \infty} \frac{x}{2x} = \frac{1}{2}$
 double x , double y
 $y = x, y = x^2 \quad \lim_{x \rightarrow \infty} \frac{x}{x^2} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$
 $y = (2x)^2 = 4x^2$
 double x , quadruple y
 x^2 faster than x
 x slower than x^2

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Which function grows faster?

$$e^x, x^2$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^2} = \lim_{x \rightarrow \infty} \frac{e^x}{2x} = \lim_{x \rightarrow \infty} \frac{e^x}{2} = \infty$$

So e^x faster than x^2

$$\ln x, x^2$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$\ln x$ slower

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Show the functions grow at the same rate

$$x, x + \sin x$$

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x} = \lim_{x \rightarrow \infty} \left(1 + \frac{\sin x}{x}\right) = \left(1 + \frac{1}{x}\right) = 1$$

$$\log_a x, \log_b x$$

$$\lim_{x \rightarrow \infty} \frac{\log_a x}{\log_b x} = \lim_{x \rightarrow \infty} \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln b}} = \frac{\ln b}{\ln a}$$

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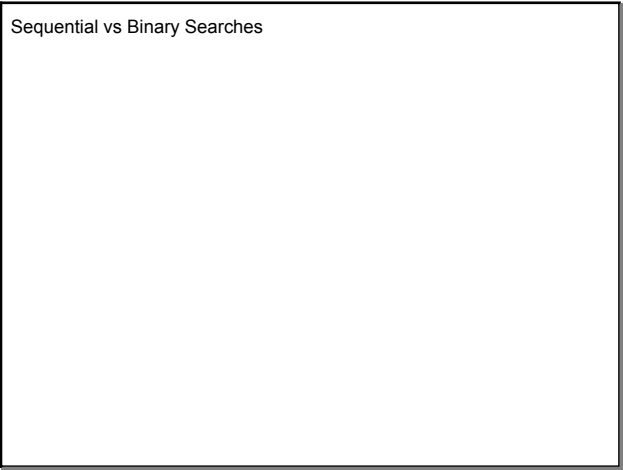
Transitivity of Growing Rates

Show the functions grow at the same rate by comparing both with x

$$\sqrt{x^2 + 5}, (2\sqrt{x} - 1)^2$$

$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5}}{x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5}}{\sqrt{x^2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5}}{x} = 1$
 $\lim_{x \rightarrow \infty} \frac{(2\sqrt{x} - 1)^2}{x} = \lim_{x \rightarrow \infty} \frac{(2\sqrt{x} - 1)^2}{x} = 4$

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